ELASTIC, ELASTIC-PLASTIC, AND VISCOELASTIC STATES OF STRESS AND STRAIN AROUND A VERTICAL CYLINDRICAL HOLE IN A SEMI-INFINITE GRAVITATIONAL BODY

BY

ALANSON DALE TOPPING

B.S., Purdue University, 1939

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AN ABSTRACT OF A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN THEORETICAL AND APPLIED MECHANICS IN THE GRADUATE COLLEGE OF THE UNIVERSITY OF ILLINOIS, 1951

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ELASTIC, ELASTIC-PLASTIC, AND VISCOELASTIC STATES OF STRESS AND STRAIN AROUND A VERTICAL CYLINDRICAL HOLE IN A SEMI-INFINITE GRAVITATIONAL BODY

The stresses and strains around a vertical cylindrical hole in a semi-infinite gravitational body are of importance in both petroleum and mining engineering because of their connection with caving and wall collapse. In this thesis, the chief objectives are (1) to establish the stresses and strains assuming that the material is elastic up to a certain limiting state of stress, beyond which it behaves plastically, without strain-hardening; and (2) to establish the stresses and strains as functions of time assuming that the material is viscoelastic. As a basis for the solution of these two problems, it is necessary to (1) obtain a rigorous solution for the stresses and strains assuming an elastic, homogeneous, isotropic material; and (2) determine the condition of plastic yielding for rock materials. It is recognized that there are many factors which influence wall collapse in an actual oil well or vertical mine shaft; and, while the matter of inelasticity is given primary attention, some other pertinent factors are discussed.

A consideration of the results of triaxial tests on rock and concrete made by a number of investigators shows that the magnitude of the shearing stress or strain at the yield limit on either the octahedral plane or the slip plane is a function not only of the normal stress on that plane but also of the direction of the shearing stress or strain in that plane (i.e., not only of the first invariant, but also of the third). Two yield criteria are developed expressing this observation mathematically. One, a generalization of the Mohr theory, reduces to the maximum shear theory of strength for materials equally strong in tension and compression; the other, a generalization of a theory advocated by Schleicher and Nadai, reduces to the distortion energy or ocahedral shearing stress theory of strength for materials equally strong in tension and compression.

The condition of fracture is shown to be a function of strain rather than of stress, and is also dependent on direc-

tion. For viscoelastic materials, it is a function of the elastic strain, and for elastic-plastic materials, a function of the elastic strain and also of the plastic strain.

From a comparison of triaxial tests on unjacketed specimens of rock and concrete with similar tests of jacketed specimens, the effective porosity of such materials is estimated and some peculiarities of stress analysis in porous media are clarified.

Mathematical solutions for the stresses and strains around a vertical cylindrical hole in a semi-infinite gravitational body are obtained by assuming (1) an elastic material; (2) an elastic-plastic material incompressible in the plastic zone; (3) an elastic-plastic material compressible in the plastic zone; (4) an elastic-plastic material having constant volumetric strain in the plastic zone; (5) several ideal compressible viscoelastic materials; (6) compressible viscoelastic materials obeying an empirical creep law. Of these, (3) requires numerical methods, but the alternate solutions (2) and (4) yield anomalous results. The elastic-plastic solutions (2), (3), and (4) all employ the first of the two yield criteria mentioned above and the Hencky or deformation theory of plasticity. In the viscoelastic solutions (5) and (6), octahedral components are used to isolate the response of the material to volumetric stress from its response to shearing stress.

The yield and fracture criteria developed are quite general, and are undoubtedly applicable to other materials which have strengths different in tension and compression, such as cast iron and ceramics. The considerations on porosity may also find application not only to other problems in mining and petroleum engineering, but in other fields, most obviously soil mechanics and geophysics. The discussion of viscoelasticity is perfectly general, and the method of solution used in the problem at hand is applicable to any other creep problem in which the stresses do not vary with time.

VITA

Alanson Dale Topping was born on March 8, 1917, in Lafayette, Indiana. After graduating from West Lafayette High School, in 1935, he entered Purdue University and received the degrees of Bachelor of Science and Bachelor of Science in Civil Engineering from that institution in 1939 and 1940, respectively. After a period of about a year as structural draftsman with the American Bridge Company, he went to the Missouri School of Mines as instructor in mechanics. He began his graduate study there, but it was interrupted by the war, and in 1944, he joined the Goodyear Aircraft Corporation as stress analyst. In 1946, he returned to the Missouri School of Mines as instructor in mechanics and in August, 1947, received the degree of Master of Science in Civil Engineering from that institution. From 1947 until 1949, he held the position of assistant professor of engineering mechanics at Purdue University. During this period he again entered into graduate study, this time in the field of mechanics. In 1949, he came to the University of Illinois to complete work toward a doctorate; there, he also acted as part time instructor in theoretical and applied mechanics. He has been a registered professional engineer in the state of Indiana since May, 1949, and is a member of Chi Epsilon, Sigma Xi, the American Society for Engineering Education, and the American Association of University Professors, and associate member of the American Society of Civil Engineers. He is author or co-author of the following publications:

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